International Journal of Engineering Sciences & Research Technology

(A Peer Reviewed Online Journal)
Impact Factor: 5.164

IJESRT

ISSN: 2277-9655
Impact Factor: 5.164
CODEN: IJESS7

IJESRT
INTERNATIONAL JOURNAL OF ENGINEERING SCIENCES & RESEARCH TECHNOLOGY

REVAN SOMBOR INDICES AND THEIR EXPONENTIALS FOR CERTAIN NANOTUBES

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DOI: 10.5281/zenodo.6623554

ABSTRACT
In this paper, we introduce the modified Revan Sombor index, Revan Sombor exponential and modified Revan Sombor exponential of a graph. We compute the Revan Sombor index, modified Revan Sombor index and their corresponding exponentials of certain nanotubes.

Keywords: *molecular structure, Revan Sombor index, modified Revan Sombor index, nanotube.*
Mathematics Subject Classification: *05C05, 05C07, 05C92.*

1. INTRODUCTION
Let G be a finite, simple, connected graph with vertex set $V(G)$ and edge set $E(G)$. The degree $d_G(u)$ of a vertex u is the number of vertices adjacent to u. Let Δ and δ denote, respectively, the maximum and minimum degree among the vertices of the graph G. The edge connecting the vertices u and v will be denoted by uv.

One of the main directions of recent research in chemical graph theory is the study and application of graph-based molecular structural descriptors, usually referred to as "topological indices" [1]. An important group of such descriptors are the vertex-degree-based (VDB) topological indices, whose general form is

$$TI = TI(G) = \sum_{uv \in E(G)} \Phi(d_G(u), d_G(v))$$

where $\Phi(x, y)$ is a pertinently chosen function satisfying the condition $\Phi(x, y) = \Phi(y, x)$. Some of the simplest, oldest, and most detailed studied VDB indices are the first and second Zagreb index

$$M_1 = M_1(G) = \sum_{uv \in E(G)} [d_G(u) + d_G(v)] \quad , \quad M_2 = M_2(G) = \sum_{uv \in E(G)} d_G(u) d_G(v)$$

Another, recently introduced group of VDB indices [2, 3] are the Sombor and Nirmala indices

$$SO = SO(G) = \sum_{uv \in E(G)} \sqrt{d_G(u)^2 + d_G(v)^2}$$

$$N(G) = \sum_{uv \in E(G)} \sqrt{d_G(u) + d_G(v)}$$

as well as the reverse Sombor index [4]

$$SO_{rev} = SO_{rev}(G) = \sum_{uv \in E(G)} \sqrt{[\Delta - d_G(u) + 1]^2 + [\Delta - d_G(v) + 1]^2}.$$

Recently, some Sombor and Nirmala indices were studied in [5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22].

Denote by $r_G(u)$ the Revan vertex degree of a vertex u in G, defined as $r_G(u) = \Delta + \delta - d_G(u)$. In 2017 [23], Kulli conceived a class of Revan-type indices, defined in analogy to the Zagreb indices as

$$R_1(G) = \sum_{uv \in E(G)} [r_G(u) + r_G(v)],$$

ISSN: 2277-9655
Impact Factor: 5.164
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$$R_2(G)=\sum_{uv\in E(G)} r_G(u)r_G(v).$$

In [24], Kulli et al. introduced the Revan Sombor index of a graph and defined it as,

$$RSO(G)=\sum_{uv\in E(G)}\sqrt{r_G(u)^2+r_G(v)^2}=\sum_{uv\in E(G)}\sqrt{[\Delta+\delta-d_G(u)]^2+[\Delta+\delta-d_G(v)]^2}.$$

The Revan Sombor exponential of a graph G is defined as

$$RSO(G,x)=\sum_{uv\in E(G)} x^{\sqrt{r_G(u)^2+r_G(v)^2}}.$$

We introduce the modified Revan Sombor index of a graph G and it is defined as

$$^{m}RSO(G)=\sum_{uv\in E(G)}\frac{1}{\sqrt{r_G(u)^2+r_G(v)^2}}.$$

We define the modified Revan Sombor exponential of a graph G as

$$^{m}RSO(G,x)=\sum_{uv\in E(G)} x^{\frac{1}{\sqrt{r_G(u)^2+r_G(v)^2}}}.$$

Recently, some Revan indices were studied in [25, 26].

In this paper, the Revan Sombor indices of $HC_5C_7[p,q]$ nanotubes, $SC_5C_7[p,q]$ nanotubes, $KTUC_4C_8[p,q]$ nanotubes, $KTUC_4C_8[p,q]$ nanotubes are computed. For nanotubes see [12] and references cited therein.

2. RESULTS FOR $HC_5C_7[p,q]$ NANOTUBES

We consider $HC_5C_7[p,q]$ nanotubes in which p is the number of heptagones in the first row and q rows of pentagones repreated alternately. The 2-D lattice ofnanotube $HC_5C_7[8,4]$ is shown in Figure 1.

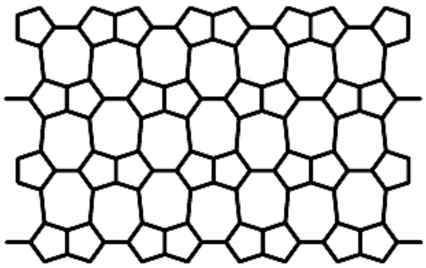

Figure 1. 2-D lattice of $HC_5C_7[8,4]$ nanotube

Let G be the graph of $HC_5C_7[p,q]$ nanotubes. We see that the vertices of G are either of degree 2 or 3. Therefore $\Delta(G) = 3$ and $\delta(G) = 2$. By algebraic method, we obtain that G has $4pq$ vertices and $6pq - p$ edges. In G, there are two types of edges based on the degree of end vertices of each edge as follows:

$E_{23}= \{uv\in E(G) \mid d_G(u) = 2, d_G(v) = 3\}$, $|E_{23}| = 4p$.
$E_{33} = \{uv\in E(G)\mid d_G(u) = d_G(v) = 3\}$, $|E_{33}| = 6pq - 5p$.

We have $r_G(u) =\Delta(G) + \delta(G) - d_G(u) = 5 - d_G(u)$.

Thus there are two types of Revan edges based on the degree of end Revan vertices of each Revan edge as given in Table 1.

Table 1: Revan edge partition of $HC_5C_7[p, q]$

$r_G(u), r_G(v) \setminus uv\in E(G)$	(3, 2)	(2, 2)
Number of edges	$4p$	$6pq - 5p$

ISSN: 2277-9655
Impact Factor: 5.164
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In the following theorem, we compute the exact formulas of $RSO(HC_5C_7[p,q])$, $RSO(HC_5C_7[p,q], x)$ for $HC_5C_7[p,q]$ nanotubes.

Theorem 1. Let G be the graph of a nanotube $HC_5C_7[p, q]$. Then

(i) $RSO\left(HC_5C_7[p,q]\right)=12\sqrt{2}pq+\left(4\sqrt{13}-10\sqrt{2}\right)p.$

(ii) $RSO\left(HC_5C_7[p,q],x\right)=4px^{\sqrt{13}}+\left(6pq-5p\right)x^{2\sqrt{2}}.$

Proof: From definitions and by using Table 1, we deduce

(i) $$RSO\left(HC_5C_7[p,q]\right)=\sum_{uv\in E(G)}\sqrt{r_G(u)^2+r_G(v)^2}=\left(3^2+2^2\right)^{\frac{1}{2}}4p+\left(2^2+2^2\right)^{\frac{1}{2}}(6pq-5p)$$
$$=12\sqrt{2}pq+\left(4\sqrt{13}-10\sqrt{2}\right)p.$$

(ii) $$RSO\left(HC_5C_7[p,q],x\right)=\sum_{uv\in E(G)}x^{\sqrt{r_G(u)^2+r_G(v)^2}}=4px^{\left(3^2+2^2\right)^{\frac{1}{2}}}+\left(6pq-5p\right)x^{\left(2^2+2^2\right)^{\frac{1}{2}}}$$
$$=4px^{\sqrt{13}}+\left(6pq-5p\right)x^{2\sqrt{2}}.$$

In the following theorem, we compute the exact formulas of ${}^{m}RSO\left(HC_5C_7[p,q]\right)$ ${}^{m}RSO\left(HC_5C_7[p,q],x\right)$ for $HC_5C_7[p,q]$ nanotubes.

Theorem 2. Let G be the graph of a nanotube $HC_5C_7[p, q]$. Then

(i) ${}^{m}RSO\left(HC_5C_7[p,q]\right)=12\sqrt{2}pq+\left(4\sqrt{13}-10\sqrt{2}\right)p.$

(ii) ${}^{m}RSO\left(HC_5C_7[p,q],x\right)=4px^{\sqrt{13}}+\left(6pq-5p\right)x^{2\sqrt{2}}.$

Proof: From definitions and by using Table 1, we deduce

(i) $${}^{m}RSO\left(HC_5C_7[p,q]\right)=\sum_{uv\in E(G)}\frac{1}{\sqrt{r_G(u)^2+r_G(v)^2}}=\frac{4p}{\sqrt{3^2+2^2}}+\frac{(6pq-5p)}{\sqrt{2^2+2^2}}$$
$$=\frac{3pq}{\sqrt{2}}+\left(\frac{4}{\sqrt{13}}-\frac{5}{2\sqrt{2}}\right)p.$$

(ii) $${}^{m}RSO\left(HC_5C_7[p,q],x\right)=\sum_{uv\in E(G)}x^{\frac{1}{\sqrt{r_G(u)^2+r_G(v)^2}}}=4px^{\frac{1}{\sqrt{3^2+2^2}}}+\left(6pq-5p\right)x^{\frac{1}{\sqrt{2^2+2^2}}}$$
$$=4px^{\frac{1}{\sqrt{13}}}+\left(6pq-5p\right)x^{\frac{1}{2\sqrt{2}}}.$$

3. RESULTS FOR $SC_5C_7[p,q]$ NANOTUBES

In this section, we focus on the family of nanotubes, denoted by $SC_5C_7[p,q]$, in which p is the number of heptagons in the first row and q rows of vertices and edges are repeated alternately. The 2-D lattice of nanotube $SC_5C_7[p,q]$ is presented in Figure 2.

ISSN: 2277-9655
Impact Factor: 5.164
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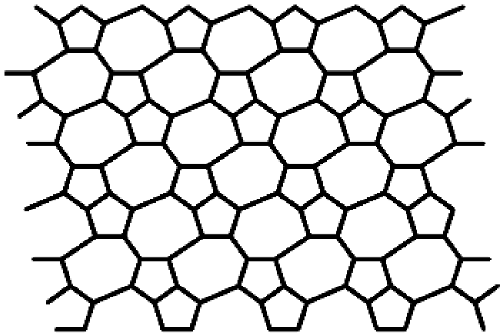

Figure 2: 2-*D* lattice of nanotube $SC_5C_7[p,q]$

Let G be the graph of $SC_5C_7[p,q]$. We see that the vertices of G are either of degree 2 or 3. Therefore $\Delta(G) = 3$ and $\delta(G) = 2$. By calculation, we obtain that G has $4pq$ vertices and $6pq - p$ edges. Also by calculation, we get that G has three types of edges based on the degree of end vertices of each edge as follows:

$E_1 = \{uv\in E(G) \mid d_G(u) = d_G(v) = 2\}$, $|E_1| = q$.
$E_2 = \{uv\in E(G) \mid d_G(u) = 2, d_G(v) = 3\}$, $|E_2| = 6q$.
$E_2 = \{uv\in E(G) \mid d_G(u) = d_G(v) = 3\}$, $|E_3| = 6pq - p – 7q$.

We have $r_G(u) = \Delta(G) + \delta(G) - d_G(u) = 5 - d_G(u)$.

Thus there are three types of Revan edges based on the degree of end Revan vertices of each Revan edge as given in Table 2.

Table 2: Revan edge partition of $SC_5C_7[p, q]$

$r_G(u), r_G(v) \setminus uv\in E(G)$	(3, 3)	(3, 2)	(2, 2)
Number of edges	q	$6q$	$6pq - p - 7q$

In the following theorem, we compute the exact formulas of $RSO(SC_5C_7[p,q])$, $RSO(SC_5C_7[p,q], x)$ for $SC_5C_7[p,q]$ nanotubes.

Theorem 3. Let G be the graph of a nanotube $SC_5C_7[p, q]$. Then

(i) $RSO(SC_5C_7[p,q]) = 12\sqrt{2}pq - 2\sqrt{2}p + (6\sqrt{13} - 11\sqrt{2})q.$

(ii) $RSO(SC_5C_7[p,q],x) = qx^{3\sqrt{2}} + 6qx^{\sqrt{13}} + (6pq - p - 7q)x^{2\sqrt{2}}.$

Proof: From definitions and by using Table 2, we derive

(i) $$RSO(SC_5C_7[p,q]) = \sum_{uv\in E(G)} \sqrt{r_G(u)^2 + r_G(v)^2}$$

$$= (3^2+3^2)^{\frac{1}{2}} q + (3^2+2^2)^{\frac{1}{2}} 6q + (2^2+2^2)^{\frac{1}{2}} (6pq - p - 7q)$$
$$= 12\sqrt{2}pq - 2\sqrt{2}p + (6\sqrt{13} - 11\sqrt{2})q.$$

(ii) $$RSO(SC_5C_7[p,q],x) = \sum_{uv\in E(G)} x^{\sqrt{r_G(u)^2 + r_G(v)^2}} = qx^{(3^2+3^2)^{\frac{1}{2}}} + 6qx^{(3^2+2^2)^{\frac{1}{2}}} + (6pq - p - 7p)x^{(2^2+2^2)^{\frac{1}{2}}}$$

$$= qx^{3\sqrt{2}} + 6qx^{\sqrt{13}} + (6pq - p - 7q)x^{2\sqrt{2}}.$$

In the following theorem, we compute the exact formulas of ${}^m RSO(SC_5C_7[p,q])$ ${}^m RSO(SC_5C_7[p,q],x)$ for $SC_5C_7[p,q]$ nanotubes.

Theorem 4. Let G be the graph of a nanotube $SC_5C_7[p, q]$.Then

(i) $^{m}RSO\left(SC_5C_7[p,q]\right) = 12\sqrt{2}pq + \left(4\sqrt{13} - 10\sqrt{2}\right)p.$

(ii) $^{m}RSO\left(SC_5C_7[p,q],x\right) = 4px^{\sqrt{13}} + \left(6pq - 5p\right)x^{2\sqrt{2}}.$

Proof: From definitions and by using Table 2, we deduce

(i) $$^{m}RSO\left(SC_5C_7[p,q]\right) = \sum_{uv\in E(G)} \frac{1}{\sqrt{r_G(u)^2 + r_G(v)^2}}$$

$$= \frac{q}{\sqrt{3^2+3^2}} + \frac{6q}{\sqrt{3^2+2^2}} + \frac{(6pq - p - 7q)}{\sqrt{2^2+2^2}}$$

$$= \frac{3pq}{\sqrt{2}} - \frac{p}{2\sqrt{2}} + \left(\frac{1}{3\sqrt{2}} + \frac{6}{\sqrt{13}} - \frac{7}{2\sqrt{2}}\right)q.$$

(ii) $$^{m}RSO\left(SC_5C_7[p,q],x\right) = \sum_{uv\in E(G)} x^{\frac{1}{\sqrt{r_G(u)^2 + r_G(v)^2}}}$$

$$= qx^{\frac{1}{\sqrt{3^2+3^2}}} + 6qx^{\frac{1}{\sqrt{3^2+2^2}}} + \left(6pq - p - 7q\right)x^{\frac{1}{\sqrt{2^2+2^2}}}$$

$$= qx^{\frac{1}{3\sqrt{2}}} + 6qx^{\frac{1}{\sqrt{13}}} + \left(6pq - p - 7q\right)x^{\frac{1}{2\sqrt{2}}}.$$

4. RESULTS FOR KTUC$_4$C$_8$[p,q] NANOTUBES

In this section, we focus on the graph structure of a family of $TUC_4C_8(S)$ nanotubes. The 2-D lattice of $TUC_4C_8(S)$ is denoted by $KTUC_4C_8[p,q]$, where q is the number of columns and q is the number of rows. The graph of $KTUC_4C_8[p,q]$ is shown in Figure 3.

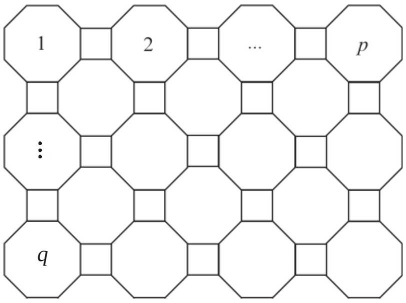

Figure 3: The graph of $KTUC_4C_8[p,q]$ nanotube

Let G be the graph of a nanotube $KTUC_4C_8[p,q]$. We see that the vertices of G are either of degree 2 or 3. Therefore $\Delta(G) = 3$ and $\delta(G) = 2$. By calculation, we obtain that G has $12pq - 2p - 2q$ edges. The graph G has three types of edges based on the degree of end vertices of each edge as follows:

$E_1 = \{uv\in E(G) \mid d_G(u) = d_G(v) = 2\}$, $|E_1| = 2p + 2q + 4$.
$E_2 = \{uv\in E(G) \mid d_G(u) = 2, d_G(v) = 3\}$, $|E_2| = 4p + 4q - 8$.
$E_3 = \{uv\in E(G) \mid d_G(u) = d_G(v) = 3\}$, $|E_3| = 12pq - 8p - 8q + 4$.

We have $r_G(u) = \Delta(G) + \delta(G) - d_G(u) = 5 - d_G(u)$.

Thus there are three types of Revan edges based on the degree of end Revan vertices of each Revan edge as given in Table 3.

Table 3: Revan edge partition of $KTUC_4C_8[p, q]$

$r_G(u), r_G(v) / uv \in E(G)$	(3, 3)	(3, 2)	(2, 2)
Number of edges	$2p+2q+4$	$4p+4q-8$	$12pq - 8p - 8q+4$

In the following theorem, we compute the exact formulas of $RSO(KTUC_4C_8[p,q])$, $RSO(KTUC_4C_8[p,q], x)$ for $KTUC_4C_8[p,q]$ nanotubes.

Theorem 5. Let G be the graph of a nanotube $KTUC_4C_8[p,q]$. Then

(i) $$RSO(KTUC_4C_8[p,q]) = 24\sqrt{2}pq + \left(4\sqrt{13} - 14\sqrt{2}\right)p + \left(4\sqrt{13} - 10\sqrt{2}\right)q + 4\sqrt{2} - 8\sqrt{13}.$$

(ii) $$RSO(KTUC_4C_8[p,q], x) = (2p+2q+4)x^{3\sqrt{2}} + (4p+4q-8)x^{\sqrt{13}} + (12pq - 8p - 8q + 4)x^{2\sqrt{2}}.$$

Proof: From definitions and by using Table 3, we obtain

(i) $$RSO(KTUC_4C_8[p,q]) = \sum_{uv \in E(G)} \sqrt{r_G(u)^2 + r_G(v)^2}$$

$$= (3^2+3^2)^{\frac{1}{2}}(2p+2q+4) + (3^2+2^2)^{\frac{1}{2}}(4p+4q-8) + (2^2+2^2)^{\frac{1}{2}}(12pq-8p-8q-4)$$

$$= 24\sqrt{2}pq + \left(4\sqrt{13} - 14\sqrt{2}\right)p + \left(4\sqrt{13} - 10\sqrt{2}\right)q + 4\sqrt{2} - 8\sqrt{13}.$$

(ii) $$RSO(KTUC_4C_8[p,q], x) = \sum_{uv \in E(G)} x^{\sqrt{r_G(u)^2 + r_G(v)^2}}$$

$$= (2p+2q+4)x^{(3^2+3^2)^{\frac{1}{2}}} + (4p+4q-8)x^{(3^2+2^2)^{\frac{1}{2}}} + (12pq-8p-8q+4)x^{(2^2+2^2)^{\frac{1}{2}}}.$$

$$= (2p+2q+4)x^{3\sqrt{2}} + (4p+4q-8)x^{\sqrt{13}} + (12pq-8p-8q+4)x^{2\sqrt{2}}.$$

In the following theorem, we compute the exact formulas of $^{m}RSO(KTUC_4C_8[p,q])$, $^{m}RSO(KTUC_4C_8[p,q], x)$ for $KTUC_4C_8[p,q]$ nanotubes.

Theorem 6. Let G be the graph of a nanotube $KTUC_4C_8[p,q]$. Then

(i) $$^{m}RSO(KTUC_4C_8[p,q]) = \frac{6pq}{\sqrt{2}} + \left(\frac{2}{3\sqrt{2}} + \frac{4}{\sqrt{13}} - \frac{4}{\sqrt{2}}\right)p + \left(\frac{2}{3\sqrt{2}} + \frac{4}{\sqrt{13}} - \frac{4}{\sqrt{2}}\right)q + \left(\frac{4}{3\sqrt{2}} - \frac{8}{\sqrt{13}} - \frac{2}{\sqrt{2}}\right).$$

(ii) $$^{m}RSO(KTUC_4C_8[p,q], x) = (2p+2q+4)x^{\frac{1}{3\sqrt{2}}} + (4p+4q-8)x^{\frac{1}{\sqrt{13}}} + (12pq-8p-8q+4)x^{\frac{1}{2\sqrt{2}}}.$$

Proof: From definitions and by using Table 3, we obtain

(i) $$^{m}RSO(KTUC_4C_8[p,q]) = \sum_{uv \in E(G)} \frac{1}{\sqrt{r_G(u)^2 + r_G(v)^2}}$$

$$= \frac{(2p+2q+4)}{\sqrt{3^2+3^2}} + \frac{(4p+4q-8)}{\sqrt{3^2+2^2}} + \frac{(12pq-8p-8q-4)}{\sqrt{2^2+2^2}}$$

$$= \frac{6pq}{\sqrt{2}} + \left(\frac{2}{3\sqrt{2}} + \frac{4}{\sqrt{13}} - \frac{4}{\sqrt{2}}\right)p + \left(\frac{2}{3\sqrt{2}} + \frac{4}{\sqrt{13}} - \frac{4}{\sqrt{2}}\right)q + \left(\frac{4}{3\sqrt{2}} - \frac{8}{\sqrt{13}} - \frac{2}{\sqrt{2}}\right).$$

(ii) $^{m}RSO(KTUC_4C_8[p,q],x)=\sum_{uv\in E(G)} x^{\frac{1}{\sqrt{r_G(u)^2+r_G(v)^2}}}$

$$=(2p+2q+4)x^{\frac{1}{\sqrt{3^2+3^2}}}+(4p+4q-8)x^{\frac{1}{\sqrt{3^2+2^2}}}+(12pq-8p-8q+4)x^{\frac{1}{\sqrt{2^2+2^2}}}.$$

$$=(2p+2q+4)x^{\frac{1}{3\sqrt{2}}}+(4p+4q-8)x^{\frac{1}{\sqrt{13}}}+(12pq-8p-8q+4)x^{\frac{1}{2\sqrt{2}}}.$$

5. RESULTS FOR $GTUC_4C_8[p,q]$ NANOTUBES

In this section, we focus on the graph structure of family of $TUC_4C_8(S)$ nanotubes. The 2-dimensional lattice of $TUC_4C_8(S)$ is denoted by $G=GTUC_4C_8[p,q]$ where $p$is the number of columns and $q$is the number of rows. The graph of$GTUC_4C_8[p,q]$ is depicted in Figure 4.

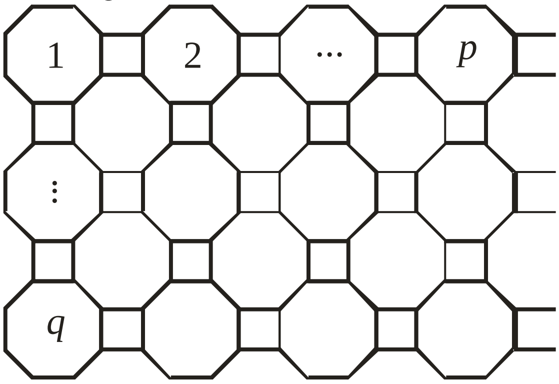

Figure 4: The graph of $GTUC_4C_8[p,q]$ nanotube

Let G be the molecular graph of $GTUC_4C_8[p,q]$ nanotube. We see that the vertices of G are either of degree 2 or 3. Therefore $\Delta(G) = 3$ and $\delta(G) = 2$. By calculation, we obtain that G has $12pq - 2p$ edges. Also by calculation, we obtain that G has three types of edges based on the degree of end vertices of each edge as follows:

$E_1 = \{uv\in E(G) \mid d_G(u) = d_G(v) = 2\}$, $|E_1| = 2p$.
$E_2 = \{uv\in E(G) \mid d_G(u) =2, d_G(v) = 3\}$, $|E_2| = 4p$.
$E_3 = \{uv\in E(G) \mid d_G(u) = d_G(v) = 3\}$, $|E_3| = 12pq - 8p$.

We have $r_G(u) =\Delta(G) + \delta(G) - d_G(u) = 5 - d_G(u)$.

Thus there are three types of Revan edges based on the degree of end Revan vertices of each Revan edge as given in Table 4.

Table 4: Revan edge partition of G

$r_G(u), r_G(u) \setminus uv \in E(G$	(3, 3)	(3, 2)	(2, 2)
Number of edges	$2p$	$4p$	$12pq - 8p$

In the following theorem, we compute the exact formulas of $RSO(GTUC_4C_8[p,q])$, $RSO(GTUC_4C_8[p,q], x)$ for $GTUC_4C_8[p,q]$ nanotubes.

Theorem 7. Let G be the graph of a nanotube $GTUC_4C_8[p,q]$. Then

(i) $RSO(GTUC_4C_8[p,q])=24\sqrt{2}pq+(4\sqrt{13}-10\sqrt{2})p.$

(ii) $RSO(GTUC_4C_8[p,q],x)==2px^{3\sqrt{2}}+4px^{\sqrt{13}}+(12pq-8p)x^{2\sqrt{2}}.$

Proof: From definitions and by using Table 4, we obtain

ISSN: 2277-9655
Impact Factor: 5.164
CODEN: IJESS7

(i) $RSO(GTUC_4C_8[p,q]) = \sum_{uv\in E(G)} \sqrt{r_G(u)^2 + r_G(v)^2}$

$$= (3^2+3^2)^{\frac{1}{2}} 2p + (3^2+2^2)^{\frac{1}{2}} 4p + (2^2+2^2)^{\frac{1}{2}} (12pq-8p)$$
$$= 24\sqrt{2}pq + \left(4\sqrt{13} - 10\sqrt{2}\right)p.$$

(ii) $RSO(GTUC_4C_8[p,q],x) = \sum_{uv\in E(G)} x^{\sqrt{r_G(u)^2 + r_G(v)^2}}$

$$= 2px^{(3^2+3^2)^{\frac{1}{2}}} + 4px^{(3^2+2^2)^{\frac{1}{2}}} + (12pq-8p)x^{(2^2+2^2)^{\frac{1}{2}}}.$$
$$= 2px^{3\sqrt{2}} + 4px^{\sqrt{13}} + (12pq-8p)x^{2\sqrt{2}}.$$

In the following theorem, we compute the exact formulas of $^{m}RSO(GTUC_4C_8[p,q])$, $^{m}RSO(GTUC_4C_8[p,q],x)$ for $GTUC_4C_8[p,q]$ nanotubes.

Theorem 8. Let G be the graph of a nanotube $GTUC_4C_8[p,q]$. Then

(i) $^{m}RSO(GTUC_4C_8[p,q]) = \frac{6pq}{\sqrt{2}} + \left(\frac{2}{3\sqrt{2}} + \frac{4}{\sqrt{13}} - \frac{4}{\sqrt{2}}\right)p.$

(ii) $^{m}RSO(GTUC_4C_8[p,q],x) = 2px^{\frac{1}{3\sqrt{2}}} + 4px^{\frac{1}{\sqrt{13}}} + (12pq-8p)x^{\frac{1}{2\sqrt{2}}}.$

Proof: From definitions and by using Table 4, we obtain

(i) $^{m}RSO(GTUC_4C_8[p,q]) = \sum_{uv\in E(G)} \frac{1}{\sqrt{r_G(u)^2 + r_G(v)^2}}$

$$= \frac{2p}{\sqrt{3^2+3^2}} + \frac{4p}{\sqrt{3^2+2^2}} + \frac{(12pq-8p)}{\sqrt{2^2+2^2}}$$
$$= \frac{6pq}{\sqrt{2}} + \left(\frac{2}{3\sqrt{2}} + \frac{4}{\sqrt{13}} - \frac{4}{\sqrt{2}}\right)p.$$

(ii) $^{m}RSO(GTUC_4C_8[p,q],x) = \sum_{uv\in E(G)} x^{\frac{1}{\sqrt{r_G(u)^2 + r_G(v)^2}}}$

$$= 2px^{\frac{1}{\sqrt{3^2+3^2}}} + 4px^{\frac{1}{\sqrt{3^2+2^2}}} + (12pq-8p)x^{\frac{1}{\sqrt{2^2+2^2}}}$$
$$= 2px^{\frac{1}{3\sqrt{2}}} + 4px^{\frac{1}{\sqrt{13}}} + (12pq-8p)x^{\frac{1}{2\sqrt{2}}}.$$

6. CONCLUSION

In this study, we have introduced the modified Revan Sombor index of a graph. The Revan Sombor and modified Revan Sombor indices and their exponentials of certain naotubes have been computed. In Medical Science, Chemical, Medical, biological pharmaceutical properties of molecular structure are essential for drug design. Their properties can be studied by the topological index calculation. .

Acknowledgement

This research is supported by IGTRC No. BNT/IGTRC/ 2022:2207:107 International Graph Theory Research Center, Banhatti 587311, India.

ISSN: 2277-9655
Impact Factor: 5.164
CODEN: IJESS7

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ISSN: 2277-9655
Impact Factor: 5.164
CODEN: IJESS7

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